RADIATION

Heat transfer by radiation usually takes place simultaneously with heat transfer by vection and conduction. The heat transfer by radiation is of much more importance at high perature levels as compared to the other two mechanisms. Direct-fired kettles, electric ters, steam boilers, rotary kiln, etc. are examples of chemical process equipments where liation is a major energy transfer mechanism.

Radiation : It refers to the transport of energy through space by electromagnetic waves.

Radiation is the mode of transport of energy in the form of electromagnetic waves through pace, at the speed of light (3×10^8 m/s).

It depends upon the electromagnetic waves as a means for the transfer of energy from a ource to a receiver.

Radiant energy is of the same nature as the ordinary visible light. It travels in straight line and it may be reflected from a surface. The electromagnetic waves with wavelength ranging fror 0.5 to 50 μm (microns) are of importance to radiant-heat transfer. [1 μm = 10^{-6} m]. Radiation of single wavelength is called monochromatic.

Thermal radiation is the energy emitted by a body entirely due to its temperature and restrict our discussion to this type of radiation.

Typical examples of heat transfer by radiation :

(i) Transfer of heat from the sun to the earth.

(ii) Heat loss from an unlagged steam pipe.

(iii) Use of energy from the sun in solar heaters.

(iv) Heating of a cold room by a radiant electric heater.

In contrast to conduction and convection, radiation heat transfer does not requ intervening medium (material or fluid) and the heat can be transmitted by a radiatior across an absolute vacuum.

Radiation is the only significant mode of energy/heat transfer when no medium is (e.g. the heat leakage through the evacuated walls of a thermos flask).

Absorptivity, Reflectivity and Transmissivity :

Any substance receives and gives off/emits energy in the form of electromagnetic waves. When energy emitted by a heated body falls on a second body (i.e., thermal radiation falling on a body), it will be partly absorbed, partly reflected and partly transmitted. It is the only absorbed energy that appears as a heat in the body.

The proportions of the incident energy that are absorbed, reflected and transmitted depend mainly on the characteristics of a receiver and temperature of the receiver (incident radiation = heat absorbed + heat transmitted + heat reflected).

The fraction of the incident radiation on a body that is absorbed by the body is known as the **absorptivity**. It may be denoted by the letter '**a**'.

The fraction of the incident radiation on a body that is reflected by the body is known as the **reflectivity**. It may be denoted by the letter '**r**'.

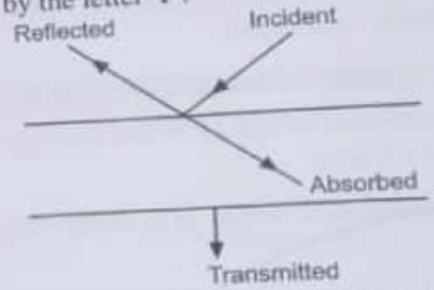


Fig. 4.1 : Reflection, absorption and transmission of radiation

The fraction of the incident radiation on a body that is transmitted through the body is known as the **transmissivity**. It may be denoted by the letter 'τ'. The energy balance about a body (a receiver) on which the total incident energy falling is unity (the sum of these fractions is unity) is given as :

$$a + r + \tau = 1.0 \quad \text{... (4.}$$

A majority of engineering materials are opaque (i.e., for which the amount transmitted very negligible, $\tau = 0$) and in such cases, the Equation (4.1) simplifies to :

$$a + r = 1.0 \text{ (as } \tau = 0) \text{ ... for an opaque material/surface ... (4}$$

If $\tau = 1$, $a = r = 0$, then all the incident energy passes through the body and it is cal perfectly transparent., e.g., rock salt (NaCl), quartz and fluorite.

If $r = 1$, $a = \tau = 0$, then all the incident energy is reflected by the body and is ca specular. If $a = 0$, $r + \tau = 1$, then the body is called as a perfectly white body, e.g., a piec white chalk (white body).

(i) $r = 0$ represents a non-reflecting surface.

(ii) $r = 1$ represents a perfect reflector.

(iii) $a = 0$ represents a non-absorbing surface.

(iv) $a = 1$ represents a perfectly absorbing surface or a black surface.

(v) $\tau = 1$ represents a perfectly transparent surface.

(vi) $\tau = 0$ represents an opaque surface.

Black Body :

A black body is an idealised physical body which absorbs all incident electromagnetic radiation. It is a perfect emitter and a perfect absorber of thermal radiation.

A body for which $a = 1$, $r = \tau = 0$, i.e., which absorbs all the incident radiant energy, is called a **black body**. It neither reflects nor transmits, but absorbs all the radiation incident on it, so it is treated as an ideal radiation receiver. It is not necessary that surface of the body be black in colour. The black body radiates maximum possible amount of energy at a given temperature and though perfectly black bodies do not exist in nature, some materials may approach it. Lampblack is the nearest to a black body. It absorbs 96 % of the visible light. Both absorptivity and emissivity of a perfectly black body are unity.

The concept of a black body is an idealisation with which the radiation characteristics of real bodies are compared.

Laws of Black Body Radiation :

Kirchhoff's Law :

This law sets up a relationship between the emissive power of a body/surface to its absorptivity.

Consider that the two bodies are kept into a furnace held at a constant temperature of T K. Assume that, of the two bodies one is a black body and the other is a non-black body, i.e., the body having 'a' value less than one. Both the bodies will ultimately attain the temperature of T K and the bodies neither become hotter nor cooler than the furnace. At this condition of thermal equilibrium, each body absorbs and emits thermal radiation at the same rate. The rate of absorption and emission for the black body will be different from that of the non-black body.

Let A_1 and A_2 be the areas of the non-black body and black body respectively. Let 'I' be the rate at which radiation falling on bodies per unit area and E_1 and E_b be the emissive powers (**emissive power is the total quantity of radiant energy emitted by a body per unit area per unit time**) of non-black and black body respectively.

At thermal equilibrium, absorption and emission rates are equal. Therefore,

$$I a_1 A_1 = A_1 E_1 \quad \dots (4.3)$$

$$\therefore \quad I a_1 = E_1 \quad \dots (4.4)$$

and

$$I a_b A_2 = A_2 E_b \quad \dots (4.5)$$

$$I a_b = E_b \quad \dots (4.6)$$

From Equations (4.4) and (4.6), we get

$$\frac{E_1}{a_1} = \frac{E_b}{a_b} \quad \dots (4.7)$$

where a_1, a_b are the absorptivities of non-black and black bodies respectively.

If we introduce a second body (non-black), then for the second non-black body, we have :

$$I A_3 a_2 = E_2 A_3 \quad \dots (4.8)$$

$$I a_2 = E_2 \quad \dots (4.9)$$

where a_2 and E_2 are the absorptivity and emissive power of the second non-black body.

Combining Equations (4.4), (4.6) and (4.9), we get

$$\frac{E_1}{a_1} = \frac{E_2}{a_2} = \frac{E_b}{a_b} = E_b \quad \dots (4.10)$$

(As the absorptivity of the black body is 1.0)

Statement of Kirchhoff's law :

It states that : *at thermal equilibrium, the ratio of the total emissive power to its absorptivity is the same for all bodies.* Equation (4.10) is the mathematical statement of Kirchhoff's law.

The **emissivity** 'e' of any body is defined as the **ratio of the total emissive power E of the body to that of a black body E_b at the same temperature.** The emissivity depends on the temperature of the body only. The emissivity of a body is a measure of how it emits radiant energy in comparison with a black body at the same temperature.

$$e = \frac{E}{E_b} \quad \text{... (4.11)}$$

Since $\frac{E}{a}$ is constant for all bodies,

$$\frac{E}{a} = \frac{E_b}{a_b} \quad \text{... (4.12)}$$

$$e = \frac{E}{E_b} = \frac{a}{a_b} \quad \text{... (4.13)}$$

But $a_b = 1$ (for black body)

$\therefore$ $e = a$ ... (4.14)

Thus, *when any body is in thermal equilibrium with its surroundings, its emissivity and absorptivity are equal.* Equation (4.14) may be taken as the another statement of **Kirchhoff's law**.

Monochromatic emissive power : It is the radiant energy emitted from a body per unit area per unit time, per unit wavelength about the wavelength λ. It is denoted by the symbol E_λ. It has the units of $W/(m^2 \cdot \mu m)$.

Total emissive power : It is the total quantity of radiant energy of all wavelength emitted by a body per unit area per unit time. It is denoted by the symbol E. The unit of E in the SI system is W/m^2.

The emissive power, E_b, of a black surface is defined as the energy emitted by the surface per unit area per unit time.

For the entire spectrum of radiation from a surface, it is the sum of all the monochromatic radiations from the surface.

$$E = \int_0^\infty E_\lambda \, d\lambda \quad \text{... (4.15)}$$

Monochromatic emissivity : *It is the ratio of the monochromatic emissive power of a surface to that of a black surface at the same wavelength.*

$$e_\lambda = \frac{E_\lambda}{E_{b,\lambda}} \quad \text{... (4.16)}$$

Grey Body :

A body having the same value of the monochromatic emissivity at all wavelengths is called a grey body.

A grey body is the one of which emissivity is independent of wavelength.

[The adjective monochromatic indicates that the quantity being defined for a particular wavelength / single wavelength. Monochromatic property refers to a single wavelength and the total property is the sum of the monochromatic values of property. Monochromatic values are important

(i) **Steafan-Boltzmann Law :**

It states that *the total emissive power (total energy emitted per unit area per unit time) of a black body is directly proportional to the fourth power of its absolute temperature*. This law relates the total amount of radiation emitted by a body (object) to its temperature.

$$E_b \propto T^4$$

$$E_b = \sigma \cdot T^4 \quad \ldots (4.17)$$

where
T = Temperature in K
σ = Steafan-Boltzmann constant
$= 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$

If E_b is in W/m^2, T is in K, then the Steafan-Boltzmann constant has the value of $5.67 \times 10^{-8}\ W/(m.K^4)$ in the SI system.

For a non-black body,

$$\frac{E}{E_b} = e \quad \ldots (4.18)$$

$$E = e \cdot E_b \quad \ldots (4.19)$$

Combining Equations (4.17) and (4.19), we get

$$E = e \cdot \sigma \cdot T^4 \quad \ldots (4.20)$$

where 'e' is the emissivity of the non-black body.

The Steafan-Boltzmann equation is a fundamental relation for all the radiant energy transfer calculations.

(ii) **Planck's Law :**

This law gives a relationship between the monochromatic emissive power of a black body, absolute temperature and the corresponding wavelength.

$$E_{b,\lambda} = \frac{2\pi\, hc^2 \lambda^{-5}}{e^{hc/k\lambda T} - 1} \quad \ldots (4.21)$$

where $E_{b,\lambda}$ is the monochromatic emissive power of the black body / black surface, $W/(m^2 \cdot \mu m)$, h is Planck's constant, k is the Boltzmann constant, c is the speed of light, T is the absolute temperature and λ is the wavelength of radiation. The Planck's constant has the value of 6.625×10^{-34} J.s in SI.

The above equation can be written as,

$$E_{b,\lambda} = \frac{C_1 \lambda^{-5}}{(e^{C_2/\lambda T} - 1)} \quad \ldots (4.22)$$

where C_1 and C_2 are constants.

$C_1 = 3.472 \times 10^{-16}\ W \cdot m^2$ and $C_2 = 0.01439\ m \cdot K$

(iii) **Wiens Displacement Law :**

It states that the wavelength at which the maximum monochromatic emissive power is obtained (i.e., λ_{max}) is inversely proportional to the absolute temperature, or

$$T \lambda_{max} = C \quad \ldots (4.23)$$

where λ_{max} is in micrometers and T is in Kelvins, the value of constant C is equal to 2890.

This law gives a relationship between the wavelength at which maximum emissive power is obtained and the absolute temperature.

Heat Transfer by Radiation :

A body having emissivity 'e' at temperature T_1 emits the radiant energy equal to $e \sigma T_1^4$ per unit area. If the surroundings are black, none of this radiation will be reflected by them and if the surroundings are at temperature T_2, they will emit the radiation equal to σT_2^4. If a body is grey, it will absorb fraction 'e' of this energy, so that the net rate of radiant energy flow from the grey body to the black surroundings is given by the expression

$$\frac{Q}{A} = e \cdot \sigma \left(T_1^4 - T_2^4\right) \quad \ldots (4.24)$$

where 'e' = Emissivity of grey body.

T_1 = Absolute temperature of grey body

T_2 = Absolute temperature of surroundings.

The Equation (4.24) is also applicable when a heat source is small as compared to the surroundings (so that none of the heat radiated from the source is reflected to it), i.e. a body radiating to the atmosphere (in the calculation of heat loss from a steam pipe).

Concept of a Black Body :

A black body is the one which absorbs all radiation incident upon it, of whatever wave length, λ. It is an ideal body that absorbs all incident radiation energy and reflects or transmits none. This means that the black body is perfectly non-reflecting and non-transmitting. Actually no matter with **a = 1** and **τ = r = 0** exists. Even the blackest surfaces occurring in nature still have reflectivity of about 1 per cent (r = 0.01).

Hence, although a black body must be black in colour, this is not a sufficient condition. Kirchhoff, however, conceived the following possibility of making a practically perfect black body. If a hollow body is provided with only one very small opening and is held at a uniform temperature, then any beam of radiation entering through the hole is partly absorbed, and partly reflected inside. The reflected radiation will not find the outlet, but will fall again on the inside of the wall. There it will be only partly reflected (other part of it is absorbed by the walls) and so on. By such a sequence of reflections, the entering radiation will be almost absorbed by the body and an arrangement of this kind will act just as a perfectly black body as shown in Fig. 4.2.

All substances emit radiation, the quality and quantity depending upon the absolute temperature and the properties of the material composing a radiating body. It may be shown that at a given temperature, good absorbers of any particular wavelength are also good emitters of that wavelength. Therefore, since by definition, a black body is a complete radiator of all wavelengths, it is also the best possible emitter of the thermal radiation, i.e., it is a full radiator.

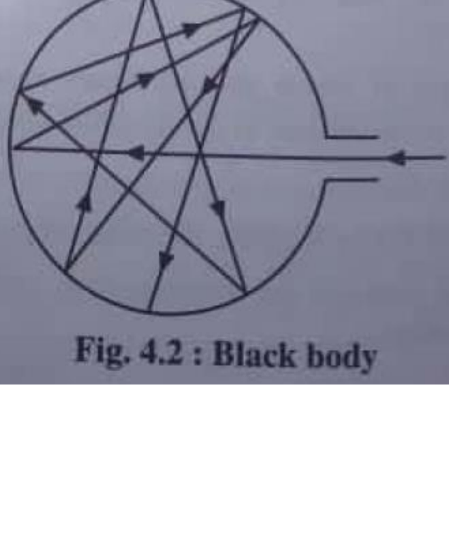


Fig. 4.2 : Black body

Transfer Coefficient for Radiation (Radiative Heat Transfer Coefficient) :

The net heat transfer by radiation from a unit surface area of a grey body at temperature T_1 to the black surroundings at temperature T_2 may be expressed as

$$Q_r = h_r (T_1 - T_2)$$

Therefore,

$$h_r = \frac{Q_r}{(T_1 - T_2)} = \frac{e \cdot \sigma}{(T_1 - T_2)} \left(T_1^4 - T_2^4\right) \quad \text{... (4.25)}$$

where h_r is the radiative heat transfer coefficient. Equation (4.21) is also applicable if the surroundings are not black, the body is small and none of its radiation is reflected back to it.

SOLVED EXAMPLES

Example 4.1 : *Calculate the heat loss by radiation from an unlagged horizontal steam pipe, 50 mm o.d. at 377 K (104°C) to air at 283 K (10°C).*

Data : Emissivity, e = 0.90.

Solution : The heat loss by radiation per unit area is given by

$$\frac{Q_r}{A} = e \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$$

where

$$e = 0.90$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\text{·K}^4)$$

$$T_1 = 377 \text{ K and } T_2 = 283 \text{ K}$$

$$\frac{Q_r}{A} = 0.90 \times 5.67 \times 10^{-8} \left(\overline{377}^4 - \overline{283}^4\right)$$

$$= \mathbf{704 \text{ W/m}^2}$$ ... **Ans.**

Example 4.2 : *Calculate the rate of heat transfer by radiation from an unlagged steam pipe, 50 mm o.d., at 393 K (120°C) to air at 293 K (20°C).*

Assume emissivity 'e' of 0.9.

Solution : Given :

$$e = 0.90$$

$$T_1 = 393 \text{ K}, \quad T_2 = 293 \text{ K}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\text{·K}^4)$$

The rate of heat transfer by radiation per unit area is

$$\frac{Q_r}{A} = e \cdot \sigma \left(T_1^4 - T_2^4\right)$$

$$= 0.90 \times 5.67 \times 10^{-8} \left(\overline{393}^4 - \overline{293}^4\right)$$

$$= \mathbf{841.2 \text{ W/m}^2}$$ ... **Ans.**

Example 4.3 : *A 50 mm i.d. iron pipe at 423 K (150°C) passes through a room in which the surroundings are at temperature of 300 K (27°C). If the emissivity of the pipe metal is 0.8, what is the net interchange of radiation energy per meter length of pipe ? The outside diameter of the pipe is 60 mm.*

Solution : Length of pipe = 1 m

$\varepsilon = 0.8$,

$T_1 = 423$ K,

$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\text{·K}^4)$$

$$T_2 = 300 \text{ K}, \qquad D_o = 60 \text{ mm} = 0.06 \text{ m}$$

Exchange of Energy between Two Parallel Plates / Planes of Different Emissivities :

Multiple Reflection Method :

When two non-black bodies are situated a small distance apart, part of the energy emitted by one body will be reflected back to it by the second body and will then be partly reabsorbed and partly reflected again. Thus the heat undergoes a series of internal reflections and absorptions.

Consider two large gray planes/surfaces that are maintained at absolute temperatures T_1 and T_2 respectively, a small distance apart and exchanging radiation. Let e_1 and e_2 be the emissivities of the surfaces.

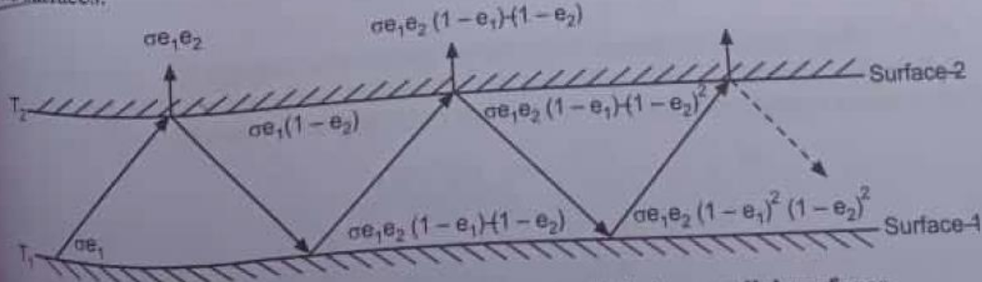


Fig. 4.3 : Radiant heat exchanger between infinite parallel surfaces (energy originating at surface-1 absorbed by surface-2)

Consider the energy radiated/emitted from the surface-1. Then, for per unit area per unit time, we have

- energy radiated from surface-1 $= \sigma \cdot e_1 T_1^4$
- of this, energy absorbed by surface-2 $= \sigma e_1 T_1^4 e_2$
- and energy reflected by surface-2 $= e_1 T_1^4 (1 - e_2)$
- of this, energy re-absorbed by surface-1 $= \sigma e_1 T_1^4 (1 - e_2) e_1$
- and energy re-reflected by surface-1 $= \sigma e_1 T_1^4 (1 - e_2)(1 - e_1)$
- and of this, energy absorbed by surface-2 $= \sigma e_1 T_1^4 (1 - e_2)(1 - e_1) e_2$

Hence, as a result of each complete cycle of internal reflection, it is clear by comparing and (f) that the absorption is reduced by a factor $(1 - e_1)(1 - e_2)$. As the energy suffers an infinite number of reflections, we can write

Total transfer of energy from surface-1 to surface-2 per unit area per unit time is

$$= \sigma \cdot e_1 e_2 T_1^4 \left[1 + (1 - e_1)(1 - e_2) + (1 - e_1)^2 (1 - e_2)^2 \ldots \text{to } \infty\right]$$

$$= \sigma \cdot e_1 e_2 T_1^4 \frac{1}{1 - (1 - e_1)(1 - e_2)}$$

$$= \frac{\sigma \cdot e_1 e_2}{e_1 + e_2 - e_1 e_2} T_1^4$$

In a similar manner, considering the radiation emitted by the surface 2, it can be shown that the total transfer of energy from surface 2 to surface 1 per unit area per unit time (i.e., energy emitted by the surface 2 and absorbed by the surface 1)

$$= \frac{e_1 e_2 \sigma}{e_1 + e_2 - e_1 e_2} T_2^4$$

Thus, the net energy transferred per unit area per unit time is

$$\left(\frac{Q}{A}\right)_{12} = \frac{e_1 e_2 \sigma}{e_1 + e_2 - e_1 e_2} \left(T_1^4 - T_2^4\right)$$

$$\left(\frac{Q}{A}\right)_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \quad \ldots(4)$$

$$\left(\frac{Q}{A}\right)_{12} = \sigma \cdot F_{12} \left(T_1^4 - T_2^4\right) \quad \ldots(4)$$

where, $$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \quad \ldots(4)$$

(F_{12} is called overall interchange factor and is function of e_1 and e_2.)

Spheres or cylinders with spherical or cylindrical enclosures : The net exchange of radiative heat or radiant energy between inner and outer spheres is given by

$$Q = \frac{\sigma A_1}{\frac{1}{e_1} + \left(\frac{r_1}{r_2}\right)^2 \left(\frac{1}{e_2} - 1\right)} \left(T_1^4 - T_2^4\right)$$

$$= \frac{\sigma A_1}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} \left(T_1^4 - T_2^4\right)$$

The net exchange of radiant energy between infinitely large concentric cylinders is given by

$$Q = \frac{\sigma \cdot A_1 (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} = \frac{\sigma \cdot A_1 (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{r_1}{r_2}\left(\frac{1}{e_2} - 1\right)} \quad \ldots (4.31)$$

where A_1 and A_2 are the areas of the inner and outer cylinders/spheres respectively, e_1 and e_2 are the emissivities of the inner and outer cylindrical/spherical surfaces. T_1 and T_2 are the respective temperatures.

$$Q = \sigma A_1 F_{12} (T_1^4 - T_2^4) \quad \ldots (4.32)$$

where

$$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} \quad \ldots (4.33)$$

Example 4.8 : *Determine the net radiant heat exchange between two parallel oxidised iron plates, placed at a distance of 25 mm having sides 3 × 3 m. The surface temperatures of two plates are 373 K (100°C) and 313 K (40°C) respectively. Emissivities of the plates are equal. Given : $e_1 = e_2 = 0.736$.*

Solution : The interchange factor is given by

$$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

$$= \frac{1}{\frac{1}{0.736} + \frac{1}{0.736} - 1} = 0.5823$$

The radiant heat exchange between two parallel planes is given by

$$Q = \sigma A F_{12} (T_1^4 - T_2^4)$$

where

$$F_{12} = 0.5823$$

$$A = 3 \times 3 = 9 \text{ m}^2$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)$$

$$T_1 = 373 \text{ K}$$

$$T_2 = 313 \text{ K}$$

$$\therefore \quad Q = 5.67 \times 10^{-8} \times 9 \times 0.5823 \times [(373)^4 - (313)^4]$$

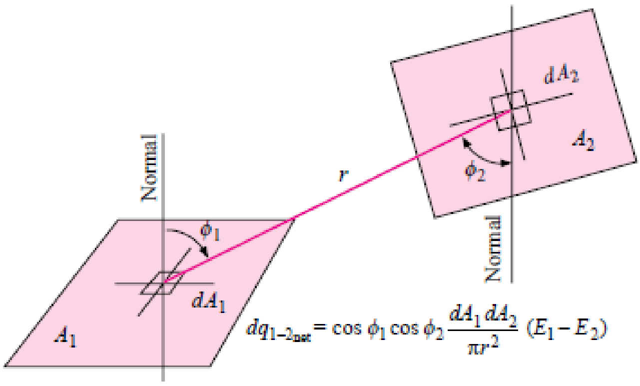
$$= \mathbf{2900\ W} \quad \ldots \text{Ans}$$

The net radiant interchange between two parallel oxidised iron plates is 2900 W.

8-4 | RADIATION SHAPE FACTOR

Consider two black surfaces A_1 and A_2, as shown in Figure 8-8. We wish to obtain a general expression for the energy exchange between these surfaces when they are maintained at different temperatures. The problem becomes essentially one of determining the amount of

Figure 8-8 | Sketch showing area elements used in deriving radiation shape factor.



energy that leaves one surface and reaches the other. To solve this problem the *radiation shape factors* are defined as

$F_{1-2} =$ fraction of energy leaving surface 1 that reaches surface 2

$F_{2-1} =$ fraction of energy leaving surface 2 that reaches surface 1

$F_{i-j} =$ fraction of energy leaving surface i that reaches surface j

Other names for the radiation shape factor are *view factor*, *angle factor*, and *configuration factor*. The energy leaving surface 1 and arriving at surface 2 is

$$E_{b1} A_1 F_{12}$$

and the energy leaving surface 2 and arriving at surface 1 is

$$E_{b2} A_2 F_{21}$$

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is

$$E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21} = Q_{1-2}$$

If both surfaces are at the same temperature, there can be no heat exchange, that is, $Q_{1-2} = 0$. Also, for $T_1 = T_2$

$$E_{b1} = E_{b2}$$

so that

$$A_1 F_{12} = A_2 F_{21} \qquad \textbf{[8-18]}$$

The net heat exchange is therefore

$$Q_{1-2} = A_1 F_{12}(E_{b1} - E_{b2}) = A_2 F_{21}(E_{b1} - E_{b2}) \qquad \textbf{[8-19]}$$

Equation (8-18) is known as a reciprocity relation, and it applies in a general way for any two surfaces i and j:

$$A_i F_{ij} = A_j F_{ji} \qquad \textbf{[8-18a]}$$

Although the relation is derived for black surfaces, it holds for other surfaces also as long as diffuse radiation is involved.

8-5 | RELATIONS BETWEEN SHAPE FACTORS

Some useful relations between shape factors may be obtained by considering the system shown in Figure 8-19. Suppose that the shape factor for radiation from A_3 to the combined area $A_{1,2}$ is desired. This shape factor must be given very simply as

$$F_{3-1,2} = F_{3-1} + F_{3-2} \quad \textbf{[8-25]}$$

that is, the total shape factor is the sum of its parts. We could also write Equation (8-25) as

$$A_3 F_{3-1,2} = A_3 F_{3-1} + A_3 F_{3-2} \quad \textbf{[8-26]}$$

and making use of the reciprocity relations

$$A_3 F_{3-1,2} = A_{1,2} F_{1,2-3}$$
$$A_3 F_{3-1} = A_1 F_{1-3}$$
$$A_3 F_{3-2} = A_2 F_{2-3}$$

the expression could be rewritten

$$A_{1,2} F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3} \quad \textbf{[8-27]}$$

which simply states that the total radiation arriving at surface 3 is the sum of the radiations from surfaces 1 and 2. Suppose we wish to determine the shape factor F_{1-3} for the surfaces in Figure 8-20 in terms of known shape factors for perpendicular rectangles with a common edge. We may write

$$F_{1-2,3} = F_{1-2} + F_{1-3}$$

in accordance with Equation (8-25). Both $F_{1-2,3}$ and F_{1-2} may be determined from Figure 8-14, so that F_{1-3} is easily calculated when the dimensions are known. Now consider the somewhat more complicated situation shown in Figure 8-21. An expression for the shape factor F_{1-4} is desired in terms of known shape factors for perpendicular rectangles

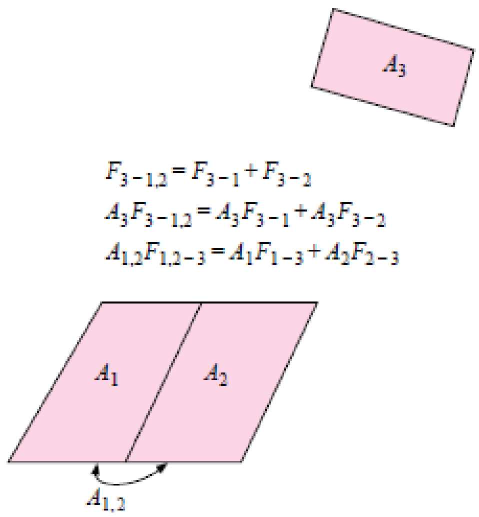


Figure 8-19 | Sketch showing some relations between shape factors.

Figure 8-20

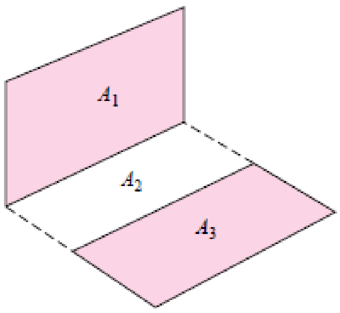
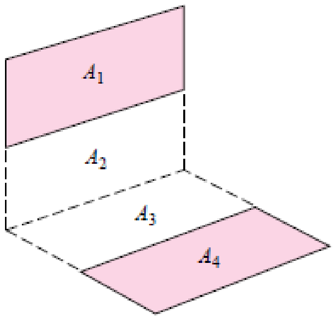


Figure 8-21



with a common edge. We write

$$A_{1,2}F_{1,2-3,4} = A_1F_{1-3,4} + A_2F_{2-3,4} \quad [a]$$

in accordance with Equation (8-25). Both $F_{1,2-3,4}$ and $F_{2-3,4}$ can be obtained from Figure 8-14, and $F_{1-3,4}$ may be expressed

$$A_1F_{1-3,4} = A_1F_{1-3} + A_1F_{1-4} \quad [b]$$

Also

$$A_{1,2}F_{1,2-3} = A_1F_{1-3} + A_2F_{2-3} \quad [c]$$

Solving for A_1F_{1-3} from (c), inserting this in (b), and then inserting the resultant expression for $A_1F_{1-3,4}$ in (a) gives

$$A_{1,2}F_{1,2-3,4} = A_{1,2}F_{1,2-3} - A_2F_{2-3} + A_1F_{1-4} + A_2F_{2-3,4} \quad [d]$$

Notice that all shape factors except F_{1-4} may be determined from Figure 8-14. Thus

$$F_{1-4} = \frac{1}{A_1}(A_{1,2}F_{1,2-3,4} + A_2F_{2-3} - A_{1,2}F_{1,2-3} - A_2F_{2-3,4}) \quad \textbf{[8-28]}$$

In the foregoing discussion the tacit assumption has been made that the various bodies do not see themselves, that is,

$$F_{11} = F_{22} = F_{33} = 0 \cdots$$

To be perfectly general, we must include the possibility of concave curved surfaces, which may then see themselves. The general relation is therefore

$$\sum_{j=1}^{n} F_{ij} = 1.0 \quad \textbf{[8-29]}$$

where F_{ij} is the fraction of the total energy leaving surface i that arrives at surface j. Thus for a three-surface enclosure we would write

$$F_{11} + F_{12} + F_{13} = 1.0$$

and F_{11} represents the fraction of energy leaving surface 1 that strikes surface 1. A certain amount of care is required in analyzing radiation exchange between curved surfaces.

Numerical 1:

Two concentric cylinders having diameters of 10 and 20 cm have a length of 20 cm. Calculate the shape factor between the open ends of the cylinders.

Solution

We use the nomenclature of Figure 8-15 for this problem and designate the open ends as surfaces 3 and 4. We have $L/r_2 = 20/10 = 2.0$ and $r_1/r_2 = 0.5$; so from Figure 8-15 or Table 8-2 we obtain

$$F_{21} = 0.4126 \qquad F_{22} = 0.3286$$

Using the reciprocity relation [Equation (8-18)] we have

$$A_1 F_{12} = A_2 F_{21} \qquad \text{and} \qquad F_{12} = (d_2/d_1) F_{21} = (20/10)(0.4126) = 0.8253$$

For surface 2 we have

$$F_{21} + F_{22} + F_{23} + F_{24} = 1.0$$

From symmetry $F_{23} = F_{24}$ so that

$$F_{23} = F_{24} = \left(\tfrac{1}{2}\right)(1 - 0.4126 - 0.3286) = 0.1294$$

Using reciprocity again,

$$A_2 F_{23} = A_3 F_{32}$$

and

$$F_{32} = \frac{\pi(20)(20)}{\pi(20^2 - 10^2)/4} 0.1294 = 0.6901$$

We observe that $F_{11} = F_{33} = F_{44} = 0$ and for surface 3

$$F_{31} + F_{32} + F_{34} = 1.0 \qquad [a]$$

So, if F_{31} can be determined, we can calculate the desired quantity F_{34}. For surface 1

$$F_{12} + F_{13} + F_{14} = 1.0$$

and from symmetry $F_{13} = F_{14}$ so that

$$F_{13} = \left(\frac{1}{2}\right)(1 - 0.8253) = 0.0874$$

Using reciprocity gives

$$A_1 F_{13} = A_3 F_{31}$$

$$F_{31} = \frac{\pi(10)(20)}{\pi(20^2 - 10^2)/4} 0.0874 = 0.233$$

Then, from Equation (a)

$$F_{34} = 1 - 0.233 - 0.6901 = 0.0769$$

Reference Books: **Heat Transfer by "J.P. Holman"**

Unit Operations in Chemical Engineering by "McCabe and smith"